EXAMINATIONS COUNCIL OF ZAMBIA

Examination for General Certificate of Education Ordinary Level

Additional Mathematics 4030/1

Paper 1

Tuesday 26 JULY 2016

Additional materials:

Answer Booklet
Graph paper (1 Sheet)
Mathematical tables/Electronic calculators (non-programmable)

Time: 2 hours

Instructions to candidates

Write your name, centre number and candidate number in the spaces on the Answer Booklet provided.

There are **12 questions** in this paper. Answer **all** questions.

Write your answers in the Answer Booklet provided.

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Information for candidates

The number of marks is shown in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

The use of a non-programmable electronic calculator is expected, where appropriate.

Cell phones are not allowed in the examination room.

You are reminded of the need for clear presentation in your answers.

Check the formulae overleaf.

MATHEMATICS FORMULAE

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots\ldots + \binom{n}{r} a^{n-r} b^r + \ldots\ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2 TRIGONOMETRY

Identities

$\text{Sin}^2 A + \cos^2 A = 1$

$\text{Sec}^2 A = 1 + \tan^2 A$

$\text{Cosec}^2 A = 1 + \cot^2 A$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The straight line $y + 3 = 2x$ meets the curve $x^2 - 3x = y^2 - 21$ at **P** and **Q**. Calculate the coordinates of **P** and **Q**. [5]

2 Find the equation of the line perpendicular to the line $2x + 4y + 3 = 0$ passing through the point $(-5, 2)$. [4]

3 Functions h and g are defined by

$h: x \to 3x - 2,$

$g: x \to \dfrac{x}{x-1},\ x \neq 1.$

Find

(a) $hg(x)$, [2]

(b) $gh(x)$, [1]

(c) the value of x for which $hg(x) = gh(x)$. [3]

4 Find the range of values of k for which the line $y = kx + 4$ does not meet the curve $2x^2 + xy = -6$. [4]

5 **(a)** Find the mid-term in the expansion of $(2 - 3x^2)^8$. [4]

(b) Find the term in x^3 in the expansion of $(1 + 2x)^3(1 - \frac{3}{2}x)^4$. [5]

6 Prove the identity

$$\frac{\mathrm{Sin}^2\theta(1-\tan^2\theta)}{\mathrm{Cos}^2\theta - \sin^2\theta} \equiv \tan^2\theta.$$ [4]

7 On the same diagram, sketch the graphs of $y = 1 + \sin 2x$ and $y = 2 - \dfrac{x}{\pi}$ for the domain 0 to 2π. Hence state the number of solutions to the equation

$1 + \sin 2x = 2 - \dfrac{x}{\pi}$. [5]

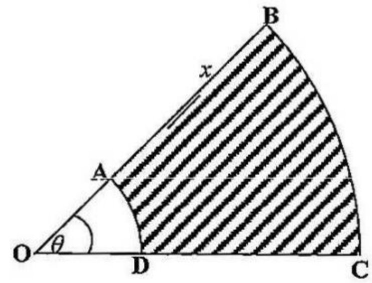
8 P and Q have position vectors $2\boldsymbol{i} - \boldsymbol{j}$ and $a\boldsymbol{i} + 4\boldsymbol{j}$ respectively. The cosine of the angle between $\underline{p}$ and $\underline{q}$ is $\dfrac{-2}{\sqrt{5}}$.

Find

(a) the value of a, [4]

(b) the unit vector in the direction of $\overrightarrow{PQ}$. [3]

9 The diagram below shows two sectors **OBC** and **OAD** with centre **O** and $\angle$**BOC** = θ radians. The radius of sector **OBC** is 3cm and **AB** = $x$cm.



(a) Show that the area of the shaded region is $\frac{1}{2}x\theta(6-x)$. [4]

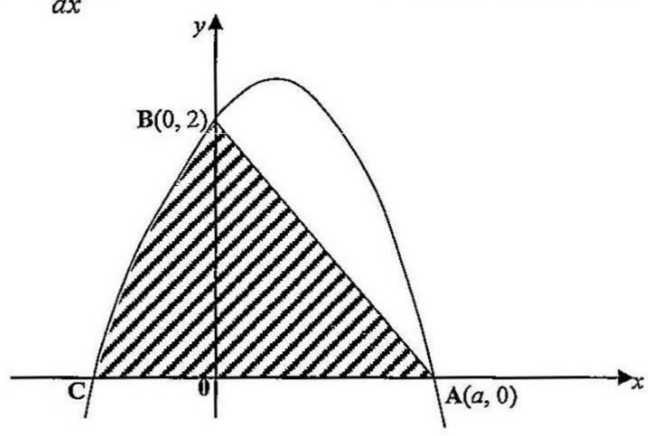
(b) Given that the area of the shaded region is 8cm^2 and $\theta = 2$ radians, find **OA**. [3]

10 (a) Find the point on the curve $y = \frac{1}{2}x^2 + \frac{8}{x}$ at which the tangent is parallel to the x-axis. [3]

(b) The radius of a cylinder is increased by 5%. Given that the height is constant, determine the percentage increase in the volume. [6]

11 (a) Find p, if $\int_p^2 (8-4x)\,dx = 18$. [3]

(b) The diagram below shows a straight line AB and a curve with a gradient function $\frac{dy}{dx} = 3 - 4x$. The curve intersects the axes at A, B and C.



Find

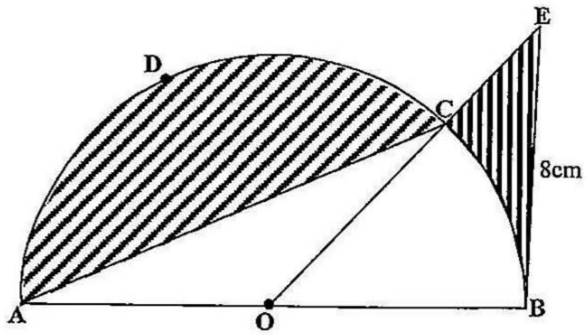
(i) the coordinates of the points **A** and **C**, [3]

(ii) the area of the shaded part. [4]

12 **Answer only one of the following alternatives:**

EITHER

In the diagram below, **ABCD** is a semi-circle with centre **O**. **AB** is the diameter and **BE** is a tangent to the circle at **B**. **BE** = 8cm and **OC**:**CE** = 3:2.



Find

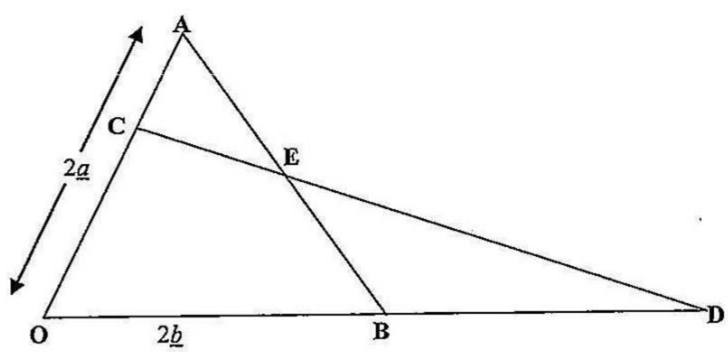
(a) $B\hat{O}E$ in radians, [2]

(b) the radius of the circle, [2]

(c) the total area of the shaded regions. [6]

OR

In the diagram below, the position vectors of **A** and **B** relative to the origin **O** are $2\underline{a}$ and $2\underline{b}$ respectively. The point **C** lies on **OA** such that **OC** = 2**CA**. The point **D** lies on **OB** produced, such that **OB** = **BD**. The lines **AB** and **CD** meet at **E**. $\overrightarrow{CE} = \lambda\overrightarrow{CD}$ and $\overrightarrow{AE} = \mu\overrightarrow{AB}$.



(a) Express $\overrightarrow{OE}$ in terms of

(i) $\underline{a}$, $\underline{b}$ and λ, [3]

(ii) $\underline{a}$, $\underline{b}$ and μ, [3]

(b) Hence or otherwise, find the value of λ and of μ. [4]